

1. $y = \frac{2}{3}(x-1)^{3/2}$ between $x=1$
and $x=2$

② length of this section

$$S = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$f(x) = y = \frac{2}{3}(x-1)^{3/2}$$

$$f'(x) = y' = \frac{dy}{dx} = \frac{3}{2} \cdot \frac{2}{3} (x-1)^{3/2-2/2} \\ = (x-1)^{1/2}$$

$$S = \int_a^b \sqrt{1 + (y')^2} dx$$

$$= \int_1^2 \sqrt{1 + ((x-1)^{1/2})^2} dx = \int_1^2 \sqrt{1 + (x-1)} dx$$

Substitution:

$$u = 1 + (x-1)$$

$$\frac{du}{dx} = 1$$

$$dx = 1 du$$

$$x=1 \rightarrow u=1$$

$$x=2 \rightarrow u=2$$

$$1 \int_{u=1}^{u=2} \sqrt{u} du = 1 \left[\frac{2}{3} u^{3/2} \right]_{u=1}^{u=2}$$

$$= 1 \left[\frac{2}{3} \cdot 2^{3/2} - \frac{2}{3} \cdot 1^{3/2} \right]$$

$$= 1 \left[\frac{2}{3} \cdot 2.828 - \frac{2}{3} \right] = 1 \left[1.885 - \frac{2}{3} \right]$$

$$= \underline{\underline{1.22...}}$$

b) Volume generated.

$$\text{Volume } V = \int_a^b \pi [f(x)]^2 dx \quad \text{OR} \quad V = \pi \int_a^b y^2 dx$$

$$= \int_1^2 \pi \left[\frac{2}{3}(x-1)^{3/2} \right]^2 dx$$

$$= \pi \int_1^2 \left(\frac{2(x-1)^{3/2}}{3} \right)^2 dx$$

$$\left(\frac{a}{b} \right)^c = \frac{a^c}{b^c} \quad \therefore \frac{(2(x-1)^{3/2})^2}{3^2}$$

$$(a \cdot b)^n = a^n b^n \quad \therefore \frac{2^2 (x-1)^{3/2 \cdot 2}}{3^2}$$

$$((x-1)^{3/2})^2 = (x-1)^3$$

$$\frac{2^2 (x-1)^3}{3^2} = \frac{4(x-1)^3}{9}$$

$$= \pi \int_1^2 \frac{4(x-1)^3}{9} dx$$

Substitution

$$u = x-1 \quad x=1 \rightarrow u=0$$

$$dx = 1 du \quad x=2 \rightarrow u=1$$

$$\pi \frac{4}{9} \int_0^1 u^3 du$$

Power rule: $\int x^a dx = \frac{x^{a+1}}{a+1} \quad a \neq -1$

$$\pi \frac{4}{9} \left[\frac{u^{3+1}}{3+1} \right]_0^1 : \frac{4\pi}{9} \left[\frac{u^4}{4} \right]_0^1$$

$$= \frac{4\pi}{9} \left[\frac{u^4}{4} \right]_0^1 : \left[\frac{u^4}{4} \right]_0^1 = \frac{1}{4}$$

$$4\pi \cdot \frac{1}{9} \cdot \frac{1}{4} = \frac{\pi}{9} = 0.34906$$

2. Points: $(3, \frac{2}{3} \ln(5) - \frac{3}{4} \ln(15))$

Slope $\frac{dy}{dx} = \frac{18-x}{12x^2-7x-12}$

Equation Expression:

Points $\frac{2}{3} \ln(5) - \frac{3}{4} \ln(15)$

~~$a \log_c(b) - \frac{3}{4} \ln(15)$~~

$a \log_c(b) = \log_c(b^a)$

$= \ln(5^{2/3}) - \ln(15^{3/4})$

$\log_c(a) - \log_c(b) = \log_c\left(\frac{a}{b}\right)$

$= \ln\left(\frac{5^{2/3}}{15^{3/4}}\right) = \frac{5^{2/3} 4\sqrt{15}}{15} = \ln\left(\frac{5^{2/3} 4\sqrt{15}}{15}\right)$

$= \ln\left(\frac{(5^{1/3})^2 (3^{1/4})}{15}\right) = 0.95807$

$(3, 0.95807)$

let the curve be $F(x, y)$ and $\frac{dy}{dx}$ be the

Slope.

Hence: $\frac{dy}{dx} = \frac{18-x}{12x^2-7x-12}$

$\int \frac{18-x}{12x^2-7x-12} dx = \int -\frac{3}{4x+3} + \frac{2}{3x-4} dx$

Sum rule: $\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$

$= -\int \frac{3}{4x+3} dx + \int \frac{2}{3x-4} dx$

$\int \frac{3}{4x+3} dx = \frac{3}{4} \ln|4x+3|$

$\int \frac{2}{3x-4} dx = \frac{2}{3} \ln|3x-4|$

$$= -\frac{3}{4} \ln|4x+3| + \frac{2}{3} \ln|3x-4|$$

$$y = -\frac{3}{4} \ln|4x+3| + \frac{2}{3} \ln|3x-4| + C$$

$$x = 3$$

$$y = 0.95807$$

$$0.95807 = -\frac{3}{4} \ln|4x+3| + \frac{2}{3} \ln|3x-4| + C$$

~~$$0.95807 = -\frac{3}{4} \ln|4x+3| + \frac{2}{3} \ln|3x-4|$$~~

$$0.95807 = \frac{3}{4} \ln|4x-3| - \frac{2}{3} \ln|3x+4| + C$$

$$C = 0.95807 + \frac{3}{4} \ln|4(3)-3| - \frac{2}{3} \ln|(3(3)+4)|$$

$$= 0.95807 + \ln(5.5^{3/4} \cdot 3^{3/4})$$

$$= 4.06206$$

$$\underline{y = -\frac{3}{4} \ln|4x+3| + \frac{2}{3} \ln|3x-4| + 4.06}$$